01 Complex Numbers I

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1. [5 marks: 1, 2, 2] [TISC]

Given the complex numbers $z_1 = 2 - i$, $z_2 = i$ and $z_3 = 2a\,i$, find:

(i) $z_1 \overline{z_2}$

(ii) $|z_1 + z_3|$

(iii) $arg\left(\frac{z_3}{2a\,z_2}\right)$.

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2. [5 marks: 1, 2, 2] [TISC]

Let the complex numbers $z_1 = a + 2i$, $z_2 = 3$ and $z_3 = \sqrt{3} - i$.

(a) Express the following in the form $x + yi$.

(i) $z_1^{\,2}$

(ii) $\frac{z_1}{z_3}$

(b) Determine in **exact form** $arg\,(i\,z_2) + arg\,(z_3)$.

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3. [7 marks: 2, 2, 3] [TISC]

Let the complex numbers $z_1 = a - 2i$, $z_2 = 1 + i$ and $z_3 = -4i$, where a is a *real number*. Determine all possible values of a if:

(a) $z_1 \times z_2 = z_3$

(b) $\frac{z_1}{z_3} = \frac{1}{2} z_2$

(c) $arg\,(z_1) + arg\,(z_2) = -\frac{\pi}{4}$.

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4. [9 marks: 2, 2, 3, 2] [TISC]

Let the complex numbers $z_1 = 2 + a\,i$ and $z_2 = 1 - 2\,i$, where a is a *real number*. Determine all possible values of a if:

(a) $z_1 \times \overline{z_1} = 2\,a^2$

(b) $z_1 = i\,\overline{z_1}$

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4. (c) $\boldsymbol{Re}\,(z_1^{\,2}) = \boldsymbol{Re}\,(z_2^{\,2})$

(d) $\boldsymbol{arg}\,(z_1) = \boldsymbol{arg}\,(\overline{z_1})$

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5. [11 marks: 2, 3, 3, 3] [TISC]

Let the complex numbers $z_1 = a + 5i$, $z_2 = 3 - 4i$ and $z_3 = 1 + i\sqrt{3}$ where a is a *real number.*

(a) Find a if $|z_1| = |z_2|$

(b) Find the exact value of a if $\tan[\boldsymbol{arg}\,(\overline{z_1})] = \tan[\boldsymbol{arg}\,(z_3)]$.

(c) Explain clearly why there is no solution for a if $|z_1 - z_2| = |z_3|$.

(d) Find the value of a if $\textbf{Im}\left(\frac{z_1}{z_2}\right) = 0$

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6. [9 marks: 3, 3, 3]

The complex number z has a modulus of 2 and an argument of $\frac{2\pi}{3}$

(a) State the complex number z^4 in Cartesian form.

(b) State the complex number $\frac{z}{i}$ in *cis* form.

(c) Given that $w \times z = 2i$, determine the complex number w.
Give your answer in polar form.

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7. [7 marks: 2, 2, 3]

Let $u = a\ cis\ \alpha$ and $v = a\ cis\ \beta$ where $a > 0$ and α and β are acute.
Express each of the following in *cis* form.

(a) $\frac{1}{u}$

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7. (b) $(\overline{v}uv)^8$

(c) $u + \overline{u}$

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8. [11 marks: 3, 4, 4] [TISC]

(a) Given that $4a - 4a\,i = r\,cis\,\theta$, find r and θ in terms of $a > 0$ where appropriate.

(b) Given that $r\,cis\left(\frac{-5\pi}{6}\right) = x + 5a\,i$, find r and x in terms of $a < 0$.

(c) Simplify $\left[\sqrt{3}\,cis\left(\frac{5\pi}{6}\right)\right]^3 \times \sqrt{3\,cis\left(\frac{\pi}{4}\right)}$

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9. [7 marks: 3, 4]

Let $x = cis\ \frac{\pi}{3}$, $y = \left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$ and $z = -1 - 3i$.

(a) Find $\frac{xy}{z-2}$ giving your answer in polar form.

(b) Find $\sqrt{3}\,x + \sqrt{2}\,y^2 + z$ giving your answer in *cis* form.

10. [7 marks: 2, 5]

(a) Express $2\sqrt{3}\ cis\left(\frac{-\pi}{3}\right)$ in Cartesian (rectangular) form.

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10. (b) Given $u + v = \sqrt{3} + i$ and $u - v = 2\sqrt{3}\, cis\left(\frac{-\pi}{3}\right)$.

Determine the complex numbers u and v giving your answer in polar form.

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11. [6 marks: 1, 2, 3]

Let $a = -1 + \sqrt{3}\, i$ and $b = -1 - i$.

(b) Find ab in exact Cartesian form.

(c) Find ab in exact cis form.

(c) Use your answers in (a) and (b) to find $\sin\left(\frac{\pi}{12}\right)$ in exact form.